Optimized Transformer Calibration

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ABSTRACT

An interesting calibration problem is presented in which one can calibrate an instrument to an uncertainty better than that offered by NIST. This is due not to any defect on the part of NIST, but to a specialized method developed for a unique application.

A specially-built, shielded and compensated voltage transformer with taps in multiples of 120 volts can be calibrated ratiometrically above 120 volts using an uncalibrated DVM of sufficient resolution. A NIST calibration is required for the 120 V tap. The interesting aspect of this ratiometric method is that the DVM does not need to be calibrated, since it is used at the same point to measure the same incremental steps. When sufficient data is accumulated, the uncertainty in the curve-fitted data reflects any errors due to repeatability in the uncalibrated DVM as well as non-linearity in the transformer. This number is added to the uncertainty in the NIST calibration at 120 V.

At the time of publication, this method only applies to the magnitude of the tap voltage ratios. Further work needs to be done to verify a null phase difference between the taps.

LEARNING OBJECTIVES

This method is of general interest to the metrology community in that it takes the basic linear curve fit used in many calibration scenarios and by applying it in a specialized application, produces better results than offered by NIST in its off-the-self calibration program. The method is elegant in that it uses a basic mathematical method optimized for maximum benefit under special circumstances.

1 -- INTRODUCTION

One of the requirements when preparing for a 17025 accreditation is to establish and document all your calibration uncertainties. This paper deals with the ratiometric calibration to within a few tens of ppm of a high-precision, high voltage actively-compensated and guarded voltage autotransformer with equally spaced taps at 120, 240, 360, and 480 Vac . The purpose for which the transformer was built is to establish NIST traceable integral multiples of 120 Volts. Multiples of 120 Volts are the most common voltages used in power generation in the United States.

There are many reasons for wanting to create such a transformer. First, calibration costs at the National Institute of Standards and Technology (NIST, an agency of the U.S. Department of Commerce) and other National Metrology Institutes tend to have a substantial proportion of the cost of calibration proportional to the number of test points required and also tend to be expensive enough that they are usually a noticeable expense in a company's metrology budget. Being able to reduce the number of calibration points required is an asset.

Second, due to the prevalence of 120 Vac and its multiples, NIST takes great pains to offer a good AC voltage calibration uncertainty at this voltage level. Unfortunately, uncertainties at voltages other than 120 Vac often decline in quality substantially. To get low uncertainties at voltages that are multiples of 120 V, a good technique is to use ratiometric techniques to multiply the original 120 V.

Finally, NIST and most other NMI's often are not prepared to perform calibration of ratio transformers or Inductive Voltage Dividers (IVD) with voltages higher than 100 Vac

However, if you have a stable, high resolution DVM you can perform the calibration of equally-spaced taps of a ratio transformer yourself in your own lab. The following calibration method checks the linearity of the taps. Once the existence of the desired linearity is established all you need to achieve precise voltages at the other taps is a precise NIST-traceable calibration on the 120 V tap.

The next question which comes up is the uncertainty of your DVM. A good DVM (6 ½ digits is probably the minimum resolution) at 120 Vac is generally accurate to ± 250 ppm -- so what good is that when you are looking for uncertainties in the tens of ppm? The answer is that as long as the DVM is stable, you don't even need to have it calibrated. You can check the linearity of the transformer by using the DVM to measure all the equal 120 V increments -- 0 to 120, 120 to 240, 240 to 360, and 360 to 480 V. When you do your linear curve fit to the data, any non-linearity in your transformer and the repeatability of your DVM will put upper limits on the quality of your calibration. If your curve fit gives you an uncertainty of ±500 ppm, that is the best you can do, and you cannot use this method to get an uncertainty to ±50 ppm. But, "the proof of the pudding is in its eating". If the curve fit gives you an uncertainty of 50 ppm, you can rely upon it.

The method will examine two different scenarios in calculating the final uncertainty. There are cases in metrology in which one is willing to trade some uncertainty for the sake of ease in calculation or use.

For the sake of illustration and simplicity, all 120 V measurements are scaled to 1 and the data are shown with wide scatter and significant non-linearity in the body of this paper. Then, in the Appendix, a new calculation is shown with real data at 120 V increments.

2 -- THE METHOD

Step1. Get a NIST-traceable calibration on the 0 to 120 Volt tap. The uncertainty will be labeled σ_1, and it needs to be less than your desired final uncertainty. This value will be used in Step 6 below.

Step 2. Take a series of measurements with the DVM from each set of transformer taps. They are in increments of 120 volts each. A set of dummy data are shown in Table 1 below. As mentioned above, 120 V data is scaled to 1 for the purposes of demonstration. The average measurements from the taps run from 1.41 to 0.5 (yes -- really poor data selected to display the effect of the method)

Table 1 -- Data taken comparing the successive 120 Volt increments on the transformer taps. All data is scaled to 1 for a 120 Volt measurement. The Max Std Dev = 0.796 Volts. This is a very conservative estimate of uncertainty due to repeatability or instability of the DVM = σ_2. (A less conservative estimate is the Ave Std Dev = 0.557 Volts)

Tap #	1	2	3	4
From tap voltage of	0	120	240	360
To tap voltage of	120	240	360	480
Voltage scaled to 1	1.286	0.667	1.394	1.031
"	0.534	1.089	0.636	0.378
"	1.892	0.435	0.432	0.339
"	1.925	2.228	0.738	0.232
Ave	1.41	1.10	0.80	0.50
StDev	0.653	0.796	0.416	0.363

Step 3. Data Analysis -- Assemble the data in a table like that shown in Table 2, below, with the x-axis composed of the tap numbers and the y-axis composed of the successive sums of the individual averaged readings. The last column shows the "Expected Values for Y". that would be measured if the taps were perfectly linear. The assembled data are illustrated in Figure 1.

Table 2 -- Assembled Calibration Data

X	Y	Composition	Expected Values for Y
0	0.000		0.000
1	1.410	Tap # 1	1.000
2	2.514	Tap # 1 + Tap # 2	2.000
3	3.314	Tap # 1 + Tap # 2 + Tap # 3	3.000
4	3.810	Continued summation	4.000

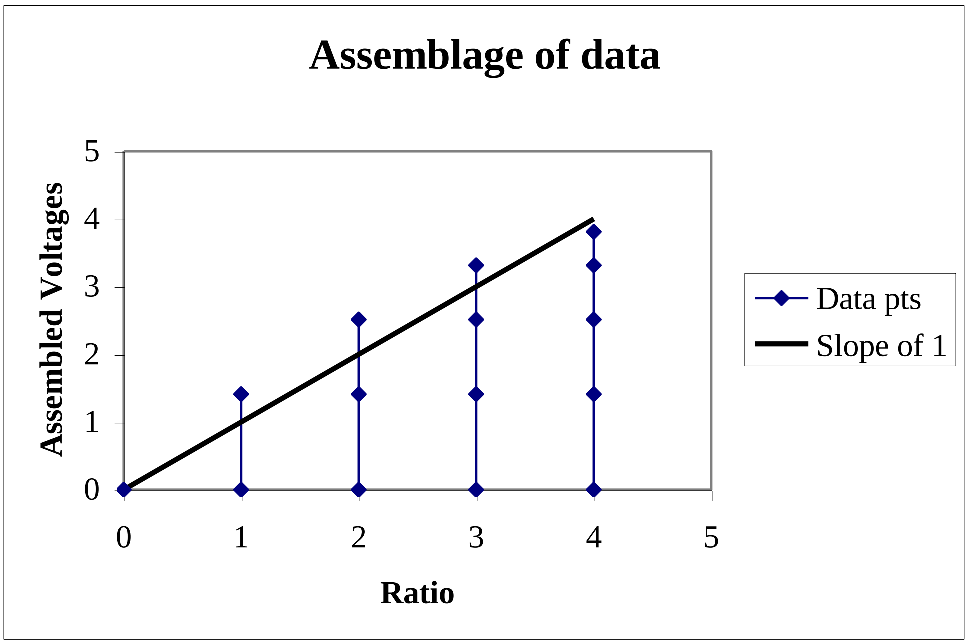

Figure 1 -- Assembled calibration data formed by combining the incremental measurements.

Step 4. Least-squares curve fit. Do the least squares curve fit to the data as shown in Table 3 and illustrated in Figure 2 with the residuals being shown in Figure 3.

Table 3 -- Least-Squares Curve Fit Data

Intercept NOT significant =	0.304 762	± 0.811 468	X ave	Y ave	No. Obs.	T	
Slope =	0.952 381	± 0.331 28	2.00	02.21	5	3.18245	
Points used in analysis (plotted points may be added further below)				95%			
X	Y	Predicted	Residuals	Interval		LCL	UCL
0	0.000	0.304 8	-0.304 8	**1.325 1**		-01.02	01.63
1	1.410	1.257 1	0.152 4	1.194 4		00.06	02.45
2	2.514	2.209 5	0.304 8	1.147 6		01.06	03.36
3	3.314	3.161 9	0.152 4	1.194 4		01.97	04.36
4	3.810	4.114 3	-0.304 8	1.325 1		02.79	05.44
Average				1.237 3			

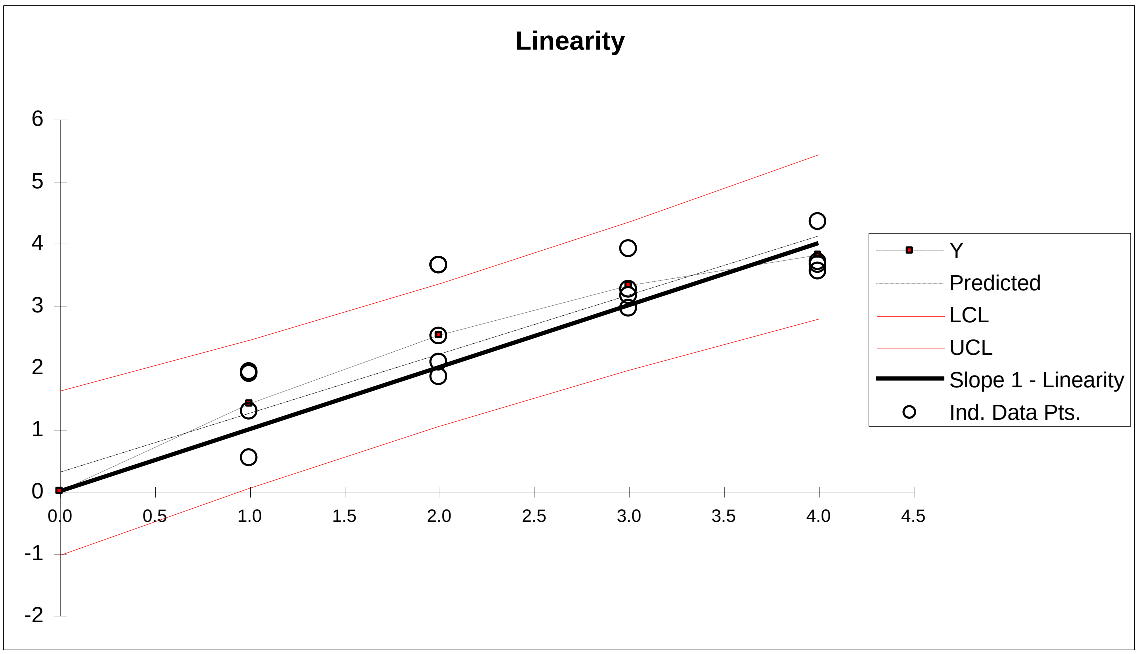

Figure 2 -- Least-squares curve fit demonstrating non-linearity and repeatability in the measuring system showing the Upper and Lower Control Limits (LCL and UCL) set at 95% confidence.

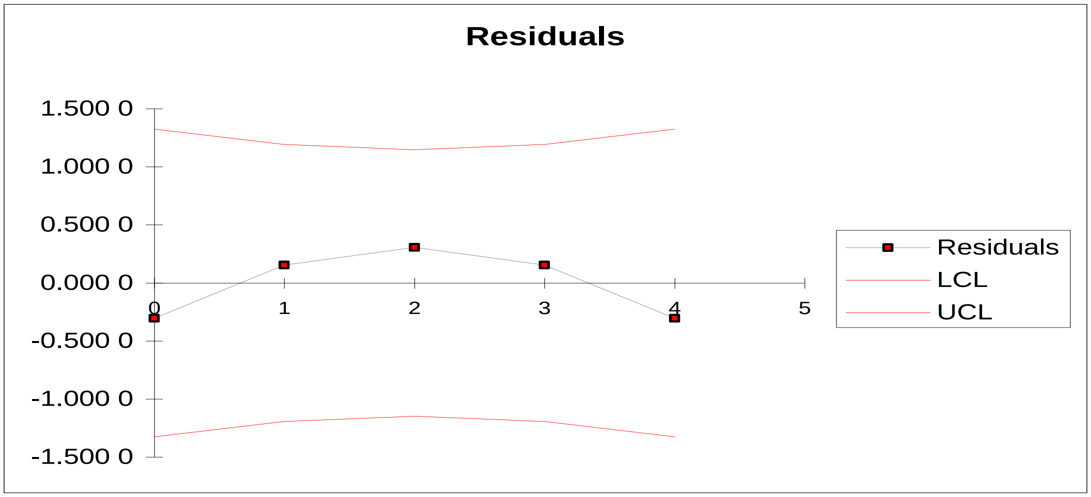

Figure 3 -- The residuals from the curve fit showing significant non-linearity

Step 5A. Choice of Uncertainty Calculation by Method A. A conservative estimate of the uncertainty due to non-linearity is the 95% confidence interval at the upper (or lower) end of the plot, ± 1.3251 Volts in this case. This is shown in the Figure 4 with a bold-face arrow and also as a bold-faced value in Table 3. The uncertainty is not easily represented as a ppm or % value, it is just a value of voltage. Since this is a 95% estimate of the uncertainty, 1/2 its value is an estimate for σ_3 which will be used in Step 6. (a less conservative estimate would be an average of the confidence intervals for all the points, 1.2373, which is not very different from the above.)

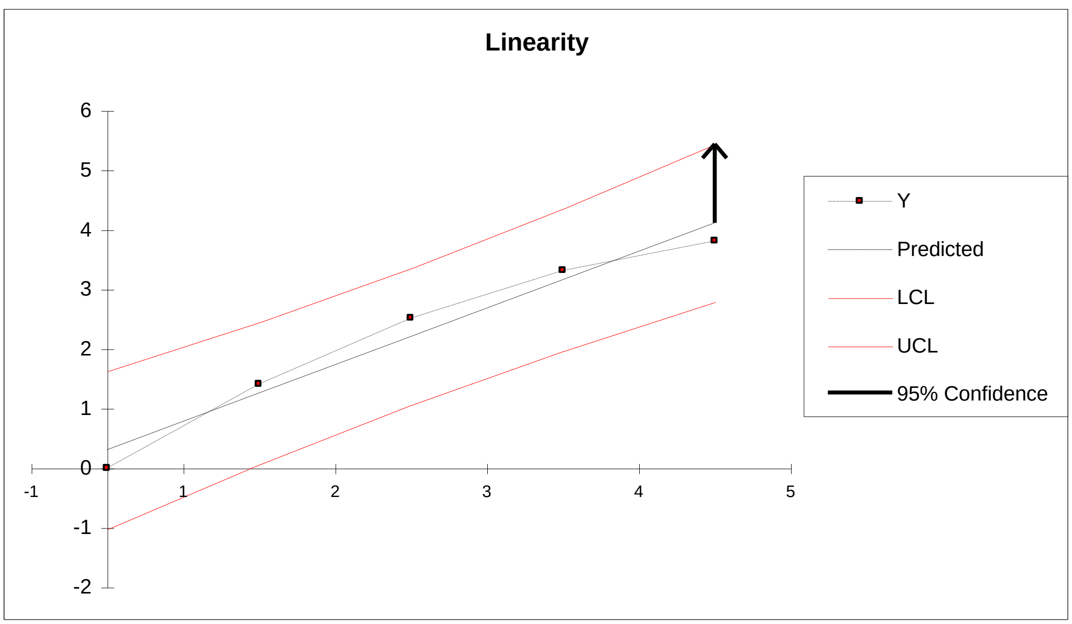

Figure 4 -- Plot of the least-squares showing the range of the 95% confidence interval.

The uncertainty calculated as an RSS combination of σ_1, σ_2 and σ_3 is the best uncertainty available treating the data as a block, but it only applies if you use the Predicted values for the ratios, which are repeated in the Table 4.

Table 4 -- Comparison of the different ratio values

X -- Same as Nominal Ratio	Y -- Same as Measured Ratio	Predicted Ratio
0	0.000	0.304 8
1	1.410	1.257 1
2	2.514	2.209 5
3	3.314	3.161 9
4	3.810	4.114 3

In other words, a technician using the transformer all day will find it much more efficient if she can calculate her results using the nominal values of 1,2,3,4, rather than the curve-fitted values of 1.2571, 2.2095, 3.1619 and 4.1143. This is only the case if the transformer has significantly different ratio values on the taps. But considering the level of accuracy required, this may be an issue. Method B, below, addresses a solution to the problem.

Step 5B. Choice of Uncertainty Calculation by Method B. Using the Nominal Ratios rather than the Predicted introduces efficiency at the risk of increased uncertainties. That will introduce an offset into the ratios which can be estimated by averaging the individual offsets of the Predicted ratio from the Nominal ratios (dashed curve to the solid curve below), shown in bold lines in the Figure 5.

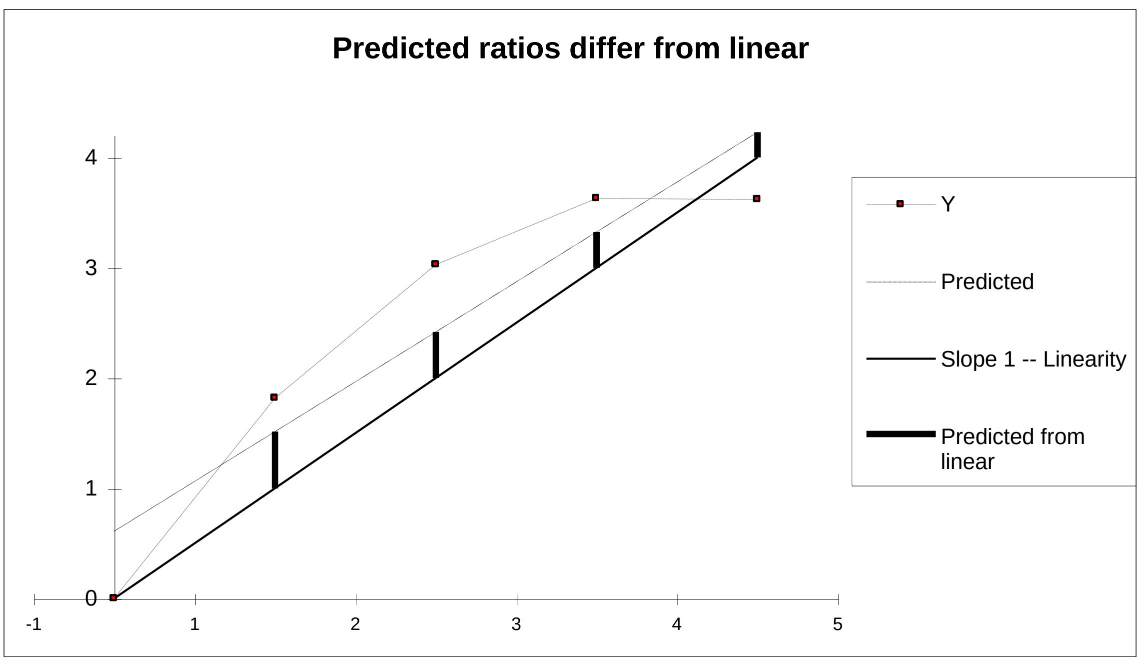

Figure 5 -- Plot of least-squares fit showing the errors between the Predicted Ratios and the Nominal Ratios (slope=1)

Table 5 -- The difference between the Nominal Ratios and the Predicted Ratios.

X	Y	Predicted	Nominal Ratio	Predicted - Nominal
1	1.41	1.2571	1	0.2571
2	2.514	2.2095	2	0.2095
3	3.314	3.1619	3	0.1619
4	3.81	4.1143	4	0.1143
			Average	0.1857
			Max	0.2095

This is a new uncertainty to add into the calculation, σ_4**.** It is the metrologist's choice whether he/she selects the maximum value (more conservative) or the average value. Since this is considered a rectangular distribution, one should divide it by the square root of 3 to obtain σ_4**.** The inclusion of an offset like this, for the sake of ease in calculation deviates from the strict G.U.M. method but it is sacrificing some uncertainty to facilitate ease of use.

Step 6. Final Uncertainty Calculation.

The combined standard uncertainty estimate consists of the RSS combination of all the components of uncertainty.

σ_1 -- uncertainty (From NIST or some other source) for the 0 to 120 V tap. As a first estimate, this would be the final standard uncertainty just for Tap #1. However, depending upon how the transformer is used and the stability of its primary power supply, it may need to be combined with σ_2.

σ_2 -- uncertainty due to instability of 6 1/2 DVM or some other source.

σ_3 -- uncertainty due to non-linearity and any additional Type A uncertainty.

σ_4 -- extra uncertainty due to offset by using nominal tap values in Method B.

σ_2 and (σ_3 and σ_4 as applicable) need to be combined with σ_1 for Tap #'s 2,3 and 4.

A general combined standard uncertainty for all taps is given below.

$$\sigma_C = \sqrt{(\sigma_1^2 + \sigma_2^2 + \sigma_3^2}$$ for a Method A Calculation

Method B deviates from proper statistical methods and in the opinion of the authors, σ_4 is best added arithmetically, since as a roughly constant offset it is less likely to cancel out with some of the other uncertainties.

$$\sigma_C = \sqrt{(\sigma_1^2 + \sigma_2^2 + \sigma_3^2} + \sigma_4$$ for a Method B Calibration.

The appropriate Expanded Combined Uncertainty is given by 2 * σ_C.

On the other hand, in opposition to Method A or Method B, one may, on a case by case basis, by sharpening one's pencil and doing significantly more mathematical work, compute more accurate uncertainties for some of the individual taps. Many times that would be excessive work for the application.

3 -- CONCLUSION

Using an uncalibrated DVM, we have presented a method to verify the linearity of equally spaced taps on a voltage transformer, which given a traceable calibration on one tap, extends the calibration to the other taps. This method only applies to the magnitude of the tap ratios. It does not extend to the phase. If the transformer presents a phase difference between the taps, the method fails to provide a traceable calibration. In order to complete the calibration, a number of experimental methods are being examined to demonstrate a null difference in phase, but the authors have not found one which meets the criterion. This will be the result of a future presentation.

APPENDIX

In the body of this paper, the calibration method was illustrated with "dummy" data that displayed significant errors that could be easily seen. In this Appendix, the calculation is presented with real data in a semi-tabulated format, below. The real transformer has an additional tap at 600 Vac.

Notes for the calculations below:

1 In the illustrated calculation in the body of this paper, all the data were scaled to 1 for convenience. In using real data, the incremental voltages were not centered around 120 volts but a slightly lower value. This could be for a variety of reasons, but since we are proposing using an uncalibrated DVM, the actual reason is not important.

   In order to establish whether the slope of the curve fit is equivalent to 1, we needed to use the average value of all the incremental measurements, 119.999 ,797 and plot the data accordingly. After the fit was performed, the slope of 119.999 857 ± 0.000 114 was not found to be significantly different from the average "Slope = 1" value of 119.999 ,797. It could be argued that use of the nominal tap voltages can be used without additional correction. However, this lack of significance could be due to a lack of sufficient degrees of freedom. For the sake of illustration and exactness, an additional uncertainty, σ_4 , will still be calculated, below, if the nominal values of the current taps are to be used, rather than the predicted values.

   To reaffirm and clarify the terminology, there are two types of linearity involved in this discussion. The first type refers to whether the data fit a straight line. In fact, there is at least some 2nd order curvature to the data. The second type refers to whether the 1st order curve fit goes through zero, i.e. if the Intercept is zero. In fact, the Intercept is significantly different from zero.

2 For the real calculation, we need a value for the repeatability. The repeatability for each level of incremental voltage is measured experimentally. However, the incremental measurements are used to verify the linearity of the transformer, but we will be using the transformer at its tap values. The repeatability from 0 to 480 V is not necessarily the repeatability of the incremental 120 V values. We could measure the repeatability at each tap, but we are not using a calibrated DVM, so the values measured would be questionable. We examined two methods of determining the repeatability for each tap in order to increase our confidence in our results. There were 11 test points at each incremental value. For the first method, we obtained 11 summed increments of the taps and calculated the standard deviation of the 11 values. In the second method, we took an RSS combination of the successive standard deviations. So, for example, the repeatability (rep) for the 360 V tap is the RSS combination of $rep_{(0-120)}$, $rep_{(120-240)}$, and $rep_{(240-360)}$. Both methods gave roughly equivalent values. We used the second method for these calculations.

Experimental Data (incremental measurements between taps)

#	**0 to 120V** (Volts)	**120 to 240V** (Volts)	**240 to 360V** (Volts)	**360 to 480V** (Volts)	**480 to 600V** (Volts)
1	119.999 ,552	119.999 ,683	119.999 ,829	119.999 ,881	120.000 ,023
2	119.999 ,552	119.999 ,690	119.999 ,829	119.999 ,866	120.000 ,031
3	119.999 ,566	119.999 ,683	119.999 ,829	119.999 ,878	120.000 ,028
4	119.999 ,570	119.999 ,697	119.999 ,836	119.999 ,885	120.000 ,016
5	119.999 ,557	119.999 ,690	119.999 ,829	119.999 ,878	120.000 ,043
6	119.999 ,549	119.999 ,697	119.999 ,829	119.999 ,866	120.000 ,038
7	119.999 ,549	119.999 ,683	119.999 ,829	119.999 ,873	120.000 ,038
8	119.999 ,557	119.999 ,690	119.999 ,841	119.999 ,863	120.000 ,045
9	119.999 ,563	119.999 ,697	119.999 ,841	119.999 ,863	120.000 ,031
10	119.999 ,542	119.999 ,690	119.999 ,841	119.999 ,873	120.000 ,031
11	119.999 ,557	119.999 ,690	119.999 ,841	119.999 ,873	120.000 ,043

Average Values

| 119.999 ,556 | 119.999 ,690 | 119.999 ,834 | 119.999 ,873 | 120.000 ,033 |

Std Deviations

| 0.000 ,008 | 0.000 ,006 | 0.000 ,006 | 0.000 ,008 | 0.000 ,009 |

Overall average to establish ideal slope (see Note 1 in the text, above, for an explanation of this number)

119.999 ,797

Successive RSS of the Std Dev's (see Note 2 in the text, above, for an explanation of these numbers)

| 0.000 ,008 | 0.000 ,010 | 0.000 ,011 | 0.000 ,014 | 0.000 ,016 |

Assembled data for the curve fit

0 to 120V	120 to 240V	240 to 360V	360 to 480V	480 to 600V
119.999 ,556	239.999 ,245	359.999 ,079	479.998 ,952	599.998 ,985

Data Analysis

X	Y	Predicted	Residuals	95% Conf. Interval	LCL	UCL
1	119.999 ,556	119.999 ,450	0.000 ,105	0.000 ,455	119.9990	119.9999
2	239.999 ,245	239.999 ,307	-0.000 ,062	0.000 ,410	239.9989	239.9997
3	359.999 ,079	359.999 ,164	-0.000 ,084	0.000 ,394	359.9988	359.9996
4	479.998 ,952	479.999 ,020	-0.000 ,068	0.000 ,410	479.9986	479.9994
5	599.998 ,985	599.998 ,877	0.000 ,109	0.000 ,455	599.9984	599.9993

Intercept = -0.000 406 ± 0.000 377. This intercept is significantly different from 0.

Slope = 119.999 857 ± 0.000 114. This slope is not significantly different from the "Slope =1 " average value found. See Note 1, above.

The results of the fit are shown below in Figure A1. As is easily seen, the transformer is so linear that the spread of points, residuals and the 95% confidence limits all merge into a single line. One needs to go to the residuals in Figure A2 to get visual resolution. The residuals show a slight curvature, which indicates a slight deviation from true linearity. This could be a true non-linearity in the transformer or a measurement artifact introduced by the high voltages in the DVM. The figure also shows that the predicted values different from the nominal tap voltages by a small amount, similar in magnitude to the residuals.

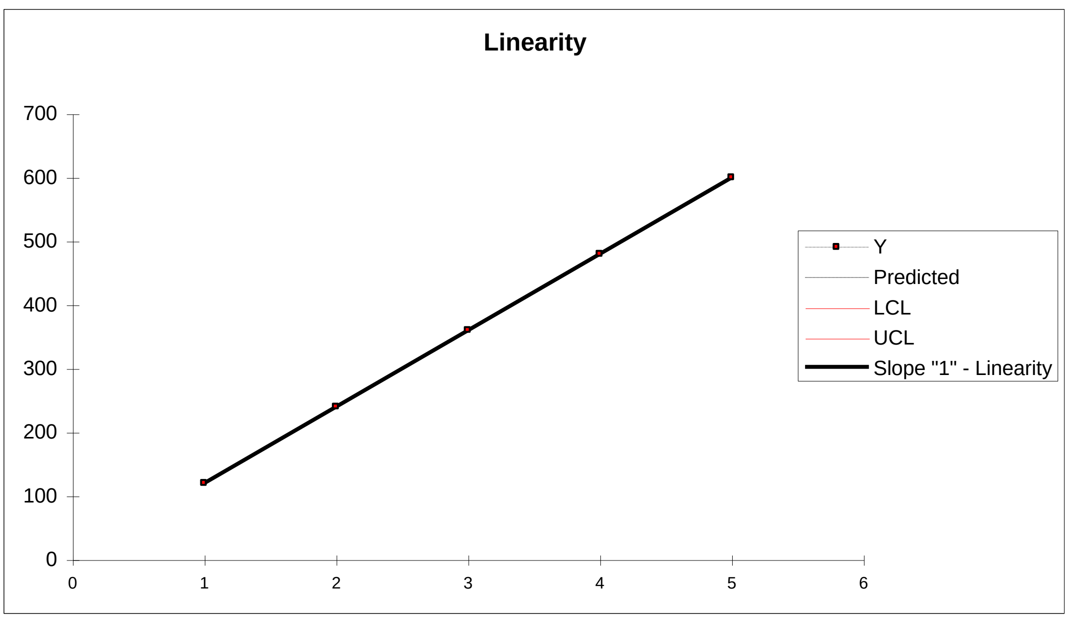

Figure A1 -- Linearity

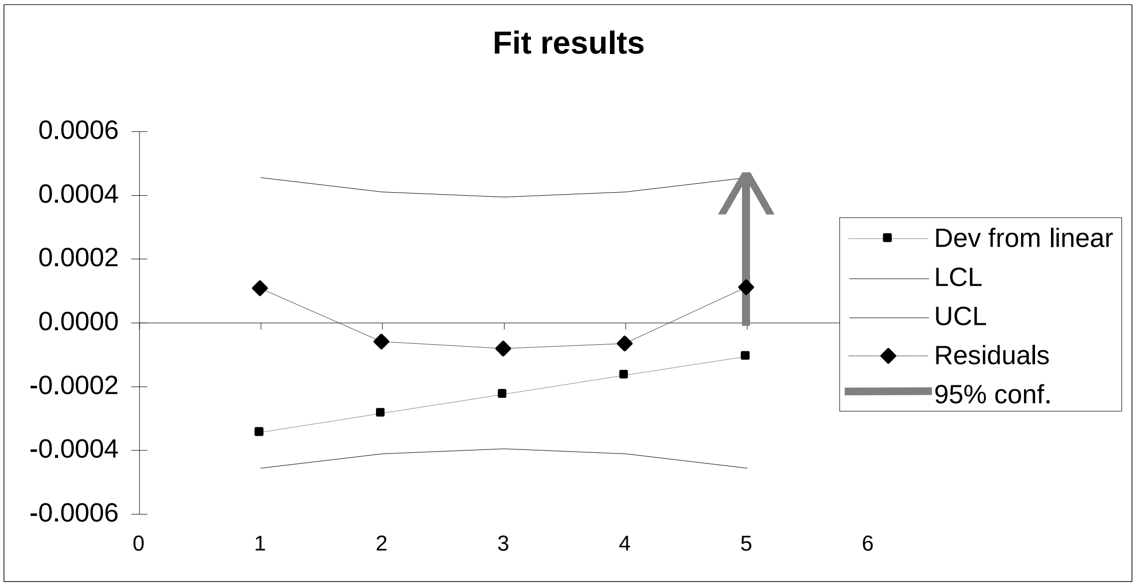

Figure A2
The "Dev from linear" curve shows the deviation from
a linear average "slope = 1" of 119.999 ,797

Components of uncertainty from the various sources

120V	**240V**	**360V**	**480V**	**600V**

σ_1 -- Calibration from NIST @ 120 V -- Standard uncertainty = 2.5 ppm

0.000 ,300	0.000 ,600	0.000 ,900	0.001 ,200	0.001 ,500

σ_2 -- Repeatability (standard deviations)

0.000 ,008	0.000 ,010	0.000 ,011	0.000 ,014	0.000 ,016

σ_3 --Uncertainty due to curve fit (including non-linearity). This value was taken from the magnitude of the 95% confidence limits from the curve fit (0.000 ,455), which was then divided by 2 to obtain a 1 sigma value.

--- *	0.000 ,228	0.000 ,228	0.000 ,228	0.000 ,228

σ_4 -- Additional uncertainty when using the nominal value of the voltage tap. The largest deviation from the nominal (straight-line) shown in Figure A1 is 0.000 ,370. This is taken to follow a uniform distribution, so it was divided by the square root of 3 to obtain a 1 sigma value.

--- *	0.000 ,234	0.000 ,234	0.000 ,234	0.000 ,234

Expanded Uncertainty by Method A in volts

$$= 2 * \sqrt{(\sigma_1^2 + \sigma_2^2 + \sigma_3^2}$$

0.000 ,600*	0.001 ,284	0.001 ,857	0.002 ,443	0.003 ,035

Expanded Uncertainty by Method A in ppm of tap value

5.0*	5.3	5.2	5.1	5.1

* See asterisk comment on next page.

* The uncertainty for the 120 V tap does not involve the curve fit, since it received the NIST calibration. Depending upon use, the NIST uncertainty may combined with the measured repeatability, σ_2.

Expanded Uncertainty by Method B in volts

$$= 2 * \left(\sqrt{(\sigma_1^2 + \sigma_2^2 + \sigma_3^2} + \sigma_4 \right)$$

0.000 ,600*	0.001 ,753	0.002 ,326	0.002 ,912	0.003 ,503

Expanded Uncertainty by Method B in ppm of tap value

5.0*	7.3	6.5	6.1	5.8

* The uncertainty for the 120 V tap does not involve the curve fit, since it received the NIST calibration. Depending upon use, the NIST uncertainty may combined with the measured repeatability, σ_2.

Concluding technical comment for those familiar with the design of amplifiers.

DVM amplifier designers will note that there are experimental questions which we have left unaddressed in this paper since the purpose is to discuss a mathematical technique rather than explain a lot of engineering details. These details concern the different high voltage fields at each tap level, grounding and CMRR's. We used a separately driven Faraday cage whose power transformer secondary was grounded to the cage. The capacitive current between the power transformer's secondary and primary was thus supplied by the separate drive and not the ratio transformer's tap. The Faraday cage was driven to keep it within 100mV of the lower voltage tap's voltage. From the DVM's point of view, its universe was constant regardless of the taps being measured.